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FURTHER STUDIES IN ESTIMATION OF PROBABILITY DISTRIBUTIONS AND --ETC(U)
JUN 82 W J PADGETT, R L TAYLOR, L J WEI AFOSR-81-0166
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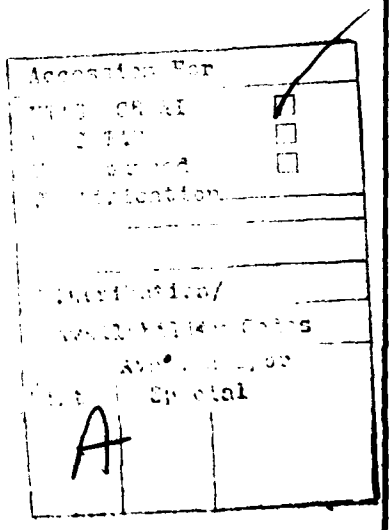
20. ABSTRACT (Continue on reverse side if necessary and identify by block number) The research papers that have been written under this grant include parametric and nonparametric estimators of survival, density, and failure rate functions for arbitrarily right-censored data as well as the independent identically distributed case. In particular, kernel estimators of density and		

failure rate functions have been investigated. Results for stochastic convergence of linear forms and randomly weighted sums of random elements in general function spaces have been obtained. These are very applicable to obtaining consistency properties of various types of statistical estimators viewed as random elements in function spaces. Nonparametric testing has also been studied, and some results in nonparametric accelerated life testing have been obtained. Statistical estimators were considered as sums of exchangeable random variables, and consistency of these estimators was obtained by the weak laws of large numbers which were developed. The particular results of the research which were obtained under this grant have attracted wide attention as evidenced by subsequent publications in journals and invited talks at professional meetings. Applications of the results should prove equally significant since maintenance policies and development of new, more reliable, equipment may be formulated using the statistical procedures and theory from these results.



I. Introduction

This document reports in detail the research results obtained and other activities of the principal investigators during the period from June 1, 1981, to May 31, 1982, under grant number AFOSR-81-0166.

In Section II a comprehensive statement of the research results obtained during this period is given. Section III contains a complete listing of approximately sixteen publications which have resulted from these research efforts. The personnel supported under this grant are listed in Section IV, and a list of professional interactions -- talks at conferences and meetings, colloquia and seminars, etc. -- is presented in Section V. Section VI gives an indication of the significance of the results to military application. Finally, other professional activities of the principal investigators are outlined in Section VII.

II. Results of the Research Effort

The research done under grant AFOSR-81-0166 during the period June 1, 1981, to May 31, 1982, has yielded a number of important results in several areas of investigation. These results can be classified into the following seven categories:

1. Nonparametric estimation of probability density functions, failure rate, and survival functions under random censorship;
2. Estimation for nonparametric and parametric models in reliability and life testing, including accelerated testing;
3. Convergence almost surely and in probability of linear forms in the space D[0,1];
4. Stochastic convergence of randomly weighted sums of random elements;
5. Nonparametric estimation and tests;
6. General stochastic systems; and
7. Laws of large numbers for exchangeable random variables.

The specific results that have been obtained in each category will be outlined below. In general, the importance of the results obtained lies in their wide applicability as well as their basic contribution to the theory of statistics and probability. The results on nonparametric maximum likelihood and kernel density estimation and kernel estimation of failure rate for arbitrarily right-censored data have attracted the attention of workers in reliability and survival analysis and apply to the assessment of quality and maintenance of equipment and manufactured items as well as to the analysis of medical data. The convergence results for random elements in function spaces provide asymptotic theory for various statistical functions which can be viewed as random elements in appropriate spaces of functions. All of the

results obtained under this grant have been well-received by the statistical community, based on the responses to talks at conferences and requests for reprints and preprints.

A detailed listing of the research papers obtained under this grant is given in Section III of this final report. We summarize here the data for grant No. AFOSR 81-0166 during the period June 1, 1981 through May 31, 1982: Three papers have been published. Six papers have been revised and accepted for publication, eight papers have been submitted for publication, eight talks have been presented at professional meetings and conferences. The principal investigators have refereed more than thirteen manuscripts for various journals and two NSF proposals during this period, as well as having reviewed seven papers and chaired two sessions at meetings. In addition, Padgett was invited to present talks at the IMS Meeting in Tallahassee, Florida, May 3-5, 1982, and at the SIAM-SEAS Meeting in Knoxville, Tennessee, on May 14-15, 1982. Taylor was invited to present a talk at the conference on limit theorems in Hungary in June, 1982, and was co-program chairman of the 1982 SREB-ASA Summer Research Conference in Statistics. Overall, this has been a very productive period for the principal investigators.

Short descriptions of the research done and results obtained in the seven general areas outlined above will be given next (papers referred to are numbers in the listing of Section III):

1. Nonparametric Estimation of Probability Density and Failure Rate Functions under Random Censorship. Four papers have been written and submitted for publication in this area (papers 4, 9, 11, and 14 in the listing in Section III). In paper 4, nonparametric maximum likelihood estimates of unimodal and decreasing probability density functions based

on arbitrarily right-censored data were obtained using techniques of order-restricted estimation and nonlinear constrained optimization. First, the density function was assumed to be decreasing (nonincreasing) on $[0,\infty)$. Then similar results were obtained for an increasing density on $[0,M]$ and a unimodal density with known or unknown mode. In all cases the maximum likelihood estimator was calculated by using a nonlinear constrained optimization algorithm. The survival function then could be obtained from the density estimate.

More general and smoother nonparametric density estimators for arbitrarily right-censored data were obtained in papers number 9 and 14 by using the kernel estimator with the empirical distribution function replaced by the usual product-limit (or Kaplan-Meier) estimate of the underlying distribution. The proportional hazards model (or Koziol-Green model) of random censorship was assumed in paper 9 in order to obtain asymptotic properties of the kernel density estimator. That is, it was assumed that there exists a nonnegative β so that $1 - F^{o}(t) = (1-H(t))^{\beta}$, where F^{o} is the underlying life distribution and H is the censoring distribution. Under this model of random censorship, it was shown that the kernel estimator is asymptotically unbiased, mean-square consistent, and weakly consistent (pointwise). These results produced very favorable reaction from workers in the area. In addition, expressions (although complicated) for the mean and variance of the kernel estimator for finite n were given. In paper number 14, for randomly censored observations a modified kernel density estimator was proposed in which the bandwidth was a function of the data. The advantage of this estimator is that the data play a role in the degree of smoothing. Convergence of the estimator in probability and almost surely to the true density was proven and an example of the estimate was presented.

For failure rate estimation, a kernel-type estimator was considered and its properties investigated under the same proportional hazards model of censorship of paper number 9. Similar unbiased and consistency results were obtained as for the density estimator. These results are contained in paper number 11 in Section III.

2. Estimation for Nonparametric and Parametric Models in Reliability and Life Testing. In this general area three papers were revised and accepted for publication in each of the journals Biometrika, IEEE Transactions on Reliability, and Canadian Journal of Statistics. In the first (paper number 1), a minimum distance estimator of the ratio of scale parameters θ, where θX and Y are assumed to have identical distributions, was obtained based on arbitrarily right-censored observations on X and Y. The two-sample Cramér—von Mises statistic was used as a distance measure between the product-limit estimators of the underlying distribution functions of a logarithmic transformation of X and Y. The resulting estimator was the median of a discrete distribution, was strongly consistent for θ, and from Monte Carlo simulations proved to be superior to other nonparametric competitors.

The second revised paper (number 3) concerned estimation under reliability growth of the reliabilities at k stages of development, assuming gamma failure models with unknown parameters. An iterative procedure involving two constrained nonlinear optimization problems was shown to converge to the maximum likelihood estimators. The solutions were relatively easy to compute using techniques of isotonic regression and tended to converge rather quickly, producing the required ordered reliability estimates for the complete samples from the k development stages. In the third paper (number 5) Bayesian lower bounds on the reliability function for the two-parameter lognormal distribution were obtained for both proper priors (inverted gamma distributions

and normal-gamma distributions) and Jeffreys' vague prior for the parameters. Monte Carlo results indicated the small sample properties and robustness of the estimators.

In paper number 6, a Bayesian procedure for estimating the mixing distribution (or prior distribution) $G(\theta)$ of a (random) parameter θ in a life distribution $F(t|\theta)$ was obtained. Based on a sample of k arbitrarily right-censored observations from the mixture $F_G(t) = \int F(t|\theta)dG(\theta)$, the estimate $\hat{G}_k$ was constructed by minimizing a measure of the distance between F_G and a nonparametric Bayes estimate of F_G. The estimate $\hat{G}_k$ of G was shown to be strongly consistent as $k \to \infty$. This paper is to appear in Utilitas Mathematica.

Finally, in paper number 15, nonparametric estimation from accelerated life tests with random censorship was considered. Let $V_1,\ldots,V_k$ denote k fixed accelereated stresses and let V denote the normal stress. It was assumed that probability distributions corresponding to the accelerated stresses differed from the nonaccelerated life distribution only by a scale factor. A simple nonparametric consistent estimator of the life distribution at the normal stress was developed for randomly right-censored data from each of the k + 1 distributions. The estimator also can be applied to accelerated life test data for items with two independent failure modes (two competing risks) at each stress level.

3. Convergence almost surely and in probability of linear forms in the space D[0,1]. One paper was written and revised for publication in this area (paper number 7). Stochastic convergence of the linear forms $\sum_{k=1}^{\infty} a_{nk} X_k$ was obtained in the space D[0,1] where $\{X_k\}$ are random elements in D[0,1] and $\{a_{nk}\}$ are real numbers. In particular, strong and weak convergence theorems were proved using boundedness conditions on the sequence of

moments and conditions on the mean oscillation of X_n on subintervals of a partition of [0,1]. Since tightness of the sequence $\{X_n\}$ is in general not implied by the hypotheses, these results represented significant improvements over previous results of Daffer and Taylor (1979), Annals of Probab., and Taylor and Daffer (1980), J. Mult. Anal., in which convex tightness (a strengthening of the concept of tightness in which all compact sets involved are also required to be convex) was used in a crucial manner to obtain laws of large numbers for random elements in $D[0,1]$. In the weak convergence results, uncorrelation conditions for certain functionals of the random elements replaced the independence assumption in obtaining the desired convergence in probability. The sharpness of these results was illustrated by example, and an investigation of the various relationships of these conditions was summarized.

4. Stochastic convergence of randomly weighted sums of random elements. In paper 9, the stochastic convergence of randomly weighted sums of random elements was obtained. In considering the empirical discrepancy over lower layers, Wright (1981) considered independent, identically distributed weights which were independent of the real-valued random variables. This assumption is also very appropriate for stratified estimates in sampling. Also, Anderson and Taylor (1976), Annals of Stat., considered least squares estimates in linear regression problems where the particular weight a_{nk} could possibly depend on the observations $X_1,\ldots,X_{k-1}$ but not on X_k. This is a natural restriction for estimates in sequential analysis. It is easy to see that the relationship between the random weights and the random elements cannot be ignored without substantially weakening the conclusions. Thus, the main results incorporated the general framework of Wright (1981), Annals of Probab., and Anderson and Taylor (1976). Moreover, the consideration

of weighted sums in Banach spaces led to applications in kernel density estimates which are weighted sums of function-valued random variables. In particular, convergence in probability and almost surely was obtained for the weighted sum $\Sigma_{k=1}^{\infty} a_{nk} X_k$ under various hypotheses involving distribution and moment conditions on the random weights and on the random elements and geometric conditions on the function space. These results contained many of the results for constant weights and real-valued random variables and extended some of the results of Howell, Taylor, and Woyczynski (1981), Proc. Probab. in Banach Spaces, III.

5. Nonparametric Tests. In this area a class of conditionally distribution-free rank order tests was developed for testing the interchangeability of two correlated variates with incomplete data on both responses (paper numbers 12, 13). It was shown that the asymptotic properties of these tests were also asymptotically distribution-free. For the class of alternative hypotheses related to possible differences in location parameters, asymptotic efficacy of the rank order tests was studied and compared with the efficacy of parametric tests for the same problem based on the bivariate normal model. Also, in paper number 2, an interval estimator of location difference in the presence of incomplete data was obtained.

6. General Stochastic Systems. In paper 10, general stochastic nonlinear feedback systems were studied with respect to existence and stability of solutions. The systems were represented by stochastic integral equations involving disturbances produced from Wiener and Poisson processes. Contractor theory of operators on Banach spaces was utilized to investigate the stability of the stochastic systems from frequency domain conditions, extending and generalizing several previous results.

7. Laws of Large Numbers for Exchangeable Random Variables. In many estimation problems, the estimator is a linear form involving possibly dependent terms which are generated from independent, identically distributed random variables. Thus, the terms are distribution symmetric under permutations, or technically, are said to be exchangeable. In paper number 16, weak laws of large numbers were obtained for exchangeable random variables.

III. Cumulative List of Written Publications for Technical Journals.

In this section, the research papers which have been written under this grant are listed. They are divided into three categories: published, accepted for publications, and submitted. Copies of the papers have already been forwarded to the Program Manager as they were submitted for publication or appeared in print, and sufficient copies of reprints will be forwarded to him as the papers appear in the future.

A. Papers Published

1. W. J. Padgett and L. J. Wei, Estimation of the ratio of scale parameters in the two sample problem with arbitrary right-censorship, Biometrika 69 (1982), 252-256.

2. L. J. Wei, Interval estimation of location difference with incomplete data, Biometrika 69 (1982), 249-251.

B. Papers Accepted for Publication

3. W. J. Padgett and D. T. McNichols, Estimation under reliability growth assuming gamma failure models, IEEE Transactions on Reliability (to appear in 1982).

4. D. T. McNichols and W. J. Padgett, Maximum likelihood estimation of unimodal and decreasing densities based on arbitrarily right-censored data, Communications in Statistics, Theory & Methods (to appear in 1982).

5. W. J. Padgett and M. P. Johnson, Some Bayesian lower bounds on reliability in the lognormal distribution, Canadian Journal of Statistics (to appear).

6. W. J. Padgett and A. N. V. Rao, Bayes estimation of a mixing or prior distribution from arbitrarily right-censored data, Utilitas Mathematica (to appear).

7. P. Z. Daffer and R. L. Taylor, Weak convergence of linear forms in D[0,1], revised for J. Multivariate Analysis.

8. R. L. Taylor and C. A. Calhoun, Almost sure convergence of randomly weighted sums of random elements, under revision for the Annals of Probability.

C. Papers Submitted for Publication

9. D. T. McNichols and W. J. Padgett, Kernel density estimation under random censorship, Statistics Tech. Rep. No. 74, October, 1981.

10. A. N. V. Rao and W. J. Padgett, Stability for a class of stochastic nonlinear feedback systems, Statistics Tech. Rep. No. 74, June, 1981.

11. D. T. McNichols and W. J. Padgett, Hazard rate estimation under the Koziol-Green model of random censorship, Statistics Tech. Rep. No. 79, Univ. of S.C., December, 1981.

12. L. J. Wei, Tests for interchangeability with incomplete paired observations, submitted.

13. L. J. Wei, Tests for independence in the presence of missing values, submitted.

14. D. T. McNichols and W. J. Padgett, A modified kernel density estimator for randomly right-censored data, Statistics Tech. Rep. No. 90, Univ. of S.C., June, 1982.

15. D. T. McNichols and W. J. Padgett, Nonparametric estimation from accelerated life tests with random censorship, Statistics Tech. Rep. No. 89, Univ. of S.C., June, 1982.

16. R. L. Taylor, Weak laws of large numbers for exchangeable random variables, Statistics Tech. Rep. No. 88, Univ. of S.C., May, 1982.

IV. Professional Personnel Associated with the Research Effort

In addition to the principal investigators, Professors Padgett, Taylor, and Wei, D. T. McNichols was a research assistant under the grant and has performed the computing for many examples used in the research papers, performed reference searches, and proofread manuscripts. This student successfully completed the Ph.D. program in May, 1982, and will be joining the faculty of the Department of Statistics at Virginia Polytechnic Institute and State University in the summer, 1982.

Additionally, during the grant period, Professor Taylor has directed one Ph.D. dissertation, Carol A. Calhoun, July, 1981, "Stochastic Convergence of Randomly Weighted Sums" and one M.S. thesis, Lynda P. Ward, December, 1981, "Statistical Analysis of Teacher Evaluation Data from the Physical Education Department of the University of South Carolina, Fall, 1980." Professor Padgett has also directed three students' M.S. theses: (i) Mary Dunlap, July, 1981, "Estimation of the Parameter and Reliability for the Rayleigh Distribution", (ii) H. Tsai, "Estimation for the Burr Failure Model", completed in March, 1982; and (iii) B. Johnson, April, 1982, "Some Approximate Prediction Intervals for the Inverse Gaussian Distribution." They also were directing one Ph.D. student and three additional masters' students in 1981-82.

V. Interactions

The principal investigators presented invited and contributed papers at various conferences. These are listed below.

(i) W. J. Padgett, invited discussant of "Statistical Catastrophe Theory" at the SREB Summer Research Conference in Statistics, De State Park. Arkansas, June 15-19, 1981.

(ii) R. L. Taylor (with C. A. Calhoun), Presented talk at the Annual Meeting of the Institute of Mathematical Statistics, Vail, Colorado, August 16-20, 1981.

(iii) L. J. Wei, Presentation at the Annual Meeting of the Institute of Mathematical Statistics, Vail, Colorado, August 16-20, 1981.

(iv) W. J. Padgett (with L. J. Wei), Presented talk at the Annual Meeting of the American Statistical Association, Detroit, Michigan, August 10-13, 1981.

(v) W. J. Padgett (with D. T. McNichols), Talk at the Special Topics Meeting on Survival Analysis, Institute of Mathematical Statistics, Columbus, Ohio, October 26-28, 1981.

(vi) R. L. Taylor, presented talk at the IMS Meeting, Tallahassee, Florida, May 3-5, 1982.

(vii) W. J. Padgett, presented an invited talk at the IMS Meeting, Tallahassee, Florida, May 3-5, 1982.

(viii) W. J. Padgett, presented an invited talk at the SIAM-SEAS Meeting in Knoxville, Tennessee, May 14-15, 1982.

In addition, Taylor gave a colloquium talk at the University of Kentucky, and Padgett addressed the Columbia Section of the ASQC in February, 1982.

VI. Inventions, Patent Disclosures, and Applications Stemming from the Research Project

No inventions or patents have stemmed from this research.

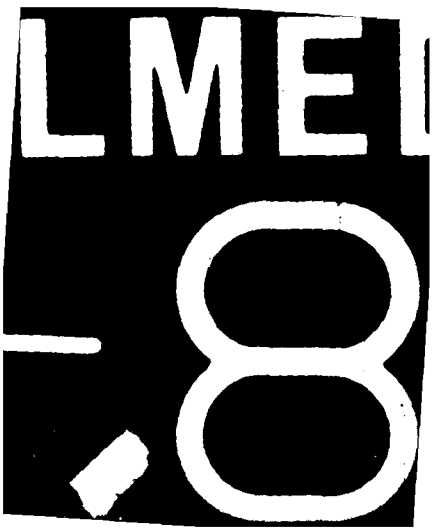
The results reported in Sections II and III have wide application in the estimation and assessment of reliability of military equipment both parametrically and nonparametrically. Thus, maintenance policies and development of new and more reliable equipment may be formulated using the statistical procedures and theory from these general results. In particular, for reliability and survival analysis, accurate and convenient estimates of survival probablities, probability density functions, and failure rate functions can be obtained only assuming general right-censorship for the observations. Also, nonparametric estimation and testing for various life distributions can be performed. Hence, assumptions of various parametric failure models are not necessary for estimation or testing for different types of equipment and experimental situations.

VII. Other Professional Activites

During this period under the grant, R. L. Taylor was Program Co-Chairman for the 1982 SREB-ASA Summer Research Conference on Statistics, and he and W. J. Padgett chaired sessions at the annual IMS and ASA meetings, respectively, in August, 1981.

The principal investigators also refereed papers for Technometrics, the Canadian Journal of Statistics, the International Journal of Mathematics and Mathematical Science, IEEE Transactions on Reliability, Communications in Statistics, Pacific Journal of Mathematics, Journal of Nonlinear Analysis TMA, the Journal of Sedimentary Petrology , and the Journal of the American Statistical Association. In addition, five reviews were written for Zentralblatt für Mathematik and two for Mathematical Reviews. Two NSF proposals were refereed.

In addition, Taylor was invited to talk at the Conference on Limit Theorems in Probability and Statistics in Hungary in June, 1982. Padgett served as an Associate Editor of the Journal of Statistical Computation and Simulation and was invited to serve on the editorial board of the new Journal of Probability and Statistics in Engineering. L. J. Wei has served on the Public Advisory Committee of NIH and was invited to talk at the SREB-ASA Summer Research Conference on Statistics in June, 1982.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